

Lower Integration Rule and Benchmark Test Procedure for Two-dimensional Reproducing Kernel Particle Method

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Abstract

Two-dimensional Reproducing Kernel Particle Method (RKPM) for large deformation analysis of mechanical structures with lower integration rule is presented. A new lower integration method, two-point quadrature for quadrilateral background mesh, is proposed to reduce the computing time. An explicit procedure for the benchmark test of the program is developed, to verify the correctness of the implementation of the formulations and the computer program. Through numerical applications, the effectiveness of the new integration method has been shown. The results obtained by the presented algorithms are compared with the analytical results.

Keywords: Meshless methods, reproducing kernel particles methods, lower integration, benchmark test, and large deformation.

Detailed Summary

Meshless methods are new generation of methods developed to overcome difficulties such as extreme material distortions in large deformation simulation. Among the meshless or meshfree methods are: the smooth particle hydrodynamics (SPH) methods [1, 3], the element-free Galerkin (EFG) method [3, 5, 6], the reproducing kernel particle method (RKPM) [1 – 4], the meshless local Petrov-Galerkin (MLPG) [6], the point interpolation method (PIM) [6], and so forth.

The RKPM, found to be very effective for large deformation analysis [2, 4], uses the finite integral representation of a function $u(\mathbf{x})$ in a domain Ω_x

$$u^a(\mathbf{x}) = \int_{\Omega_x} \Phi_a(\mathbf{x} - \mathbf{y})u(\mathbf{y})d\Omega_x \quad (1)$$

where $u^a(\mathbf{x})$ is the approximation of function $u(\mathbf{x})$, $\Phi_a(\mathbf{x} - \mathbf{x}_I)$ is the window or kernel function with compact support a .

Like the others meshless methods, the RKPM is characterized by its high CPU cost. In our implementation of RKPM the background mesh is currently used for integration purposes. The integration of the stresses for the calculation of internal forces involves several loops over the gauss quadrature points. Therefore reducing the number of gauss quadrature points, without affecting the accuracy of the results, will improve significantly the computation efficiency. In this

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work, instead of the traditional 2×2 gauss rules (called here Q4), a two gauss quadrature points are set in the integration cell as shown in **figure 1-(c)**.

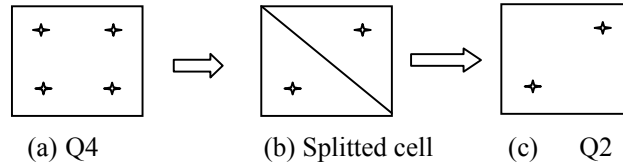


Figure 1 Integration rules

This integration rule can be derived from the 2×2 gauss quadrature integration rule by splitting the 4 node element into two triangular elements with one quadrature point as shown in **figure 1-(b)**. The resulting quadrilateral element will have two gauss quadrature points, therefore called **Q2**.

The verifications of the implementation of the computer program as well as the element formulation are extremely important in computational mechanics. For this purpose the patch test has been used in the finite element method for explicit programs using a linear field. In this work the benchmark test procedure based on an arbitrary displacement field for explicit programs is developed for two-dimensional problems based the three-dimensional developed by Li et al. [4]. While no external force is applied for the patch test in explicit programs [7], for the benchmark test the body forces are computed to satisfy the momentum equation and applied as external forces on the structures.

In this work, \mathbf{X} is the material coordinates, \mathbf{x} the spatial coordinate, and \mathbf{u} the displacements of the particles; and t the time.

In the benchmark test procedure the displacements are assumed as function of both material coordinates ($\Phi_i(\mathbf{X})$) and time ($\Psi_i(t)$) as following (no summation on $i, i = 1, 2$)

$$u_i(\mathbf{X}, t) = \Phi_i(\mathbf{X})\Psi_i(t) = f_i(\mathbf{X}, t) \quad (2)$$

In each time step equation (3.1) is implemented as boundaries values for all the essential boundary particles under the discretized form:

$$u_i(\mathbf{X}_I, t) = f_i(\mathbf{X}_I, t), \quad I = 1, \dots, NB \quad (3)$$

where NB is the total number of particles on the boundary Γ .

No external boundaries force should be prescribed.

The momentum equation is satisfied in the domain by applying the body forces calculated from the equilibrium

$$\rho b_i = \rho \ddot{u}_i - \frac{\partial P_{ji}}{\partial X_j} \quad (4)$$

The constitutive model must be linear elastic or hyper-elastic.

The benchmark test procedure is employed in the following numerical example to simulate a plane strain compression process like forging, using an isotropic linear elastic material model. The L_2 error norm is used to assess the accuracy of the integration scheme in this paper. The following mapping is used:

$$\begin{cases} x(X, Y, t) = f_1 X (Y^2 - Y_m^2) + X \\ y(X, Y, t) = f_2 Y^2 + Y \end{cases} \quad (5)$$

$$\text{Where } \begin{cases} f_1 = C_1 (\cos(\pi t / T) - 1) \\ f_2 = C_2 \sin(\pi t / T) \end{cases} \quad (6)$$

$C_1, C_2, Y_m,$ and T are parameters controlling the amplitude and the frequency of the deformation. The plane stain formulation with isotropic elastic material is used. The material parameters and values of parameters $C_1, C_2, Y_m,$ and T are given in **table 1**.

Table 1 Values of parameters controlling the deformation and material properties in 2D

Parameter	C_1 (m^{-2})	C_2 (m^{-1})	T (s)	Y_m (m)	Mass density(kg)	Young modulus (Pa)	Poisson ratio
Value	100	-2.5	10^{-3}	0.125	7.8×10^3	$22. \times 10^{10}$	0.3

A domain of 0.25×0.25 m is considered in this example. The initial configuration and the deformed shapes of the bloc are shown in **figure 2**. Convergence study is done on the integration schemes to assess the accuracy of the results in the displacement and nominal stress fields.

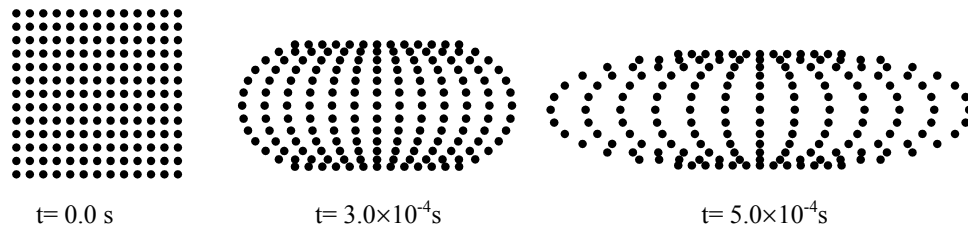


Figure 2 Initial configuration and deformed shapes

The L_2 errors in the displacements and the nominal stress are shown in the **tables 2** and **3** respectively at time $t = 5 \times 10^{-4}$ s.

Table 2 Displacement L_2 error at $t = 5 \times 10^{-4}$ s

Number of Particles	L_2 Error on the displacement %			
	Q4		Q2	
	X	Y	X	Y
5×5	9.70	12.86	9.38	11.46
9×9	2.69	4.33	2.62	4.36
13×13	1.22	2.14	1.21	2.22

It is seen that Q2 scheme is more accurate than Q4 scheme in displacement fields for coarse particles distribution.

Table 3 Nominal stress L_2 error at $t = 5 \times 10^{-4}$ s

Number of	L_2 Error on nominal stress component %	
	Q4	Q2

Particles	xx	yy	xy	yx	xx	yy	xy	yx
5×5	16.14	26.35	30.89	59.82	17.81	28.85	36.17	62.55
9×9	6.41	10.88	12.47	28.96	9.05	11.47	13.87	30.30
13×13	3.44	5.92	7.38	16.91	3.95	6.38	7.00	17.84

Conclusion

A two-point integration scheme in non-linear RKPM is proposed in this work. For the verification of the present integration scheme a general benchmark test procedure is developed for non-linear explicit program. The results obtained are clearly convergent for this integration method according to the L_2 error in the displacement and nominal stress. Using the new reduced integration method presented in this paper one can save the computing time significantly.

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